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ABSTRACT

This paper reports on the development to date of the numeracy conceptual framework for the Adult Literacy and Lifeskills (ALL) Project. The inclusion of a numeracy scale in the ALL survey offered a significant opportunity to develop a new conceptual framework for adult numeracy. This paper discusses the meaning of numeracy and the notion of numerate behavior, presents a framework of five facets, and looks in some detail at a complexity-rating scheme used to guide construction of the assessment tasks that looks at the question of what makes one numeracy task more difficult than another. (Author)

What Makes One Numeracy Task More Difficult Than Another?

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Abstract

This paper reports on the development to date of the numeracy conceptual framework for the Adult Literacy and Lifeskills (ALL) Project. The inclusion of a numeracy scale in the ALL survey offered a significant opportunity to develop a new conceptual framework for adult numeracy. This paper discusses the meaning of numeracy and the notion of numerate behavior, presents a framework of five facets, and looks in some detail at a complexity-rating scheme used to guide construction of the assessment tasks that looks at the question of what makes one numeracy task more difficult than another.

The ALL Project

An international survey of the numeracy abilities of adults is to be part of the Adult Literacy and Lifeskills (ALL) Project, formerly called the International Life Skills Survey (ILSS), planned for the year 2002. This comparative survey is being jointly developed by Statistics Canada and by the United States' National Center for Education Statistics (NCES), in cooperation with the Organization for Economic Cooperation and Development (OECD). The ALL project is a follow-up to the International Adult Literacy Survey (IALS), the world's first large-scale comparative assessment of adult literacy.

Tasks will assess performance in several skill domains, including Numeracy as well as Document and Prose Literacy, and Problem Solving, while other variables will be assessed via a background questionnaire.

To this date, items and background questions developed for the survey have undergone a feasibility study in the Netherlands and the U.S. After translation of the materials, a pilot study will be administered in the participating countries in 2001. The first round of the actual survey is scheduled for 2002.

Why Include Numeracy?

Numeracy is becoming a growing concern for diverse sectors. Countries are increasingly attending to topics such as improving workplace efficiency and quality processes, to resulting lifelong learning needs, and to civic participation. It is seen as vital that nations have information about their citizens' numeracy, among other skills, if they want to plan effective education and lifelong learning opportunities.

The concept of numeracy is also specifically related to the dialogue about the goals and especially outcomes and impact of school mathematics education. More educators now encourage links between knowledge gained in the mathematics classroom and students' ability to handle real-life situations that require mathematical or statistical knowledge and skills. However, while numeracy may be a key skill area, its conceptual boundaries, cognitive underpinnings, and assessment have not received much prior scholarly attention.

What Is Adult Numeracy?

One of the scales of the International Adult Literacy Survey (IALS), the Quantitative Literacy Scale, was a measurement of the respondent's ability to apply arithmetic operations to numbers embedded in diverse texts. While this scale produced useful data, survey developers recognized that it was limited in scope. The Numeracy scale of ALL is designed to go above and beyond the QL Scale, while avoiding reliance on formal, curriculum-based knowledge of mathematics.

IALS, following on a framework established in previous studies in the U.S. and Canada, made use of three literacy scales, Prose Literacy, Document Literacy, and Quantitative Literacy, to make operational its conception of literacy. The general definition of Literacy used in IALS was:

> Using printed and written information to function in society, to achieve one's goals, and to develop one's knowledge and potential.

The definition for Document Literacy (DL) was:

> The knowledge and skills required to locate and use information contained in various formats, including job applications, payroll forms, transportation schedules, maps, tables, and graphics.

The definition for Quantitative Literacy (QL) in IALS was:

> The knowledge and skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials.

The working definition we have for numeracy in the ALL is:

> The knowledge and skills required to effectively manage the mathematical demands of diverse situations.

Facets of Numeracy

We have sought a view of numeracy that acknowledges the diverse purposes served by adults' mathematical (and statistical) knowledge, that encompasses the different suggestions regarding the skills adults need to effectively function in home, work, community, and other contexts, and that takes into account the cognitive, metacognitive, and dispositional processes that support or affect adults' numeracy.

Overall, numeracy is a multifaceted and sometimes slippery construct. Our basic premise is that numeracy is the bridge that links mathematical knowledge, whether acquired via formal or informal learning, with functional and information-processing demands encountered in the real world. An evaluation of a person's numeracy is far from being a trivial matter, as it has to take into account task and situational demands, type of mathematical information available, the way in which that information is represented, prior practices, individual dispositions, cultural norms, and more.

A full assessment of all elements of such a broad conception is beyond the scope of ALL. We have thus chosen to focus on numerate behavior, which is revealed in the response to mathematical information that may be represented in a range of ways and forms. The nature of a person's responses to mathematical situations depends on the activation of various enabling knowledge bases, practices, and processes.

Table 1 presents our elaboration of numerate behavior. It has been used to guide development of items for a Numeracy Scale for the ALL. The statement in Table 1 distinguishes five facets, each with several components.

Table 1: Numerate Behavior and Its Facets

Numerate behavior involves:

Managing a situation or solving a problem in a real context
everyday life
work
societal
further learning

by responding
identifying or locating
acting upon
- order/sort
- count
- estimate
- compute
- measure
- model

interpreting
communicating about

to information about mathematical ideas
quantity & number
dimension & shape
pattern and relationships
data & chance
change

that is represented in a range of ways
objects & pictures
numbers & symbols
formulae
diagrams & maps
graphs
tables
texts

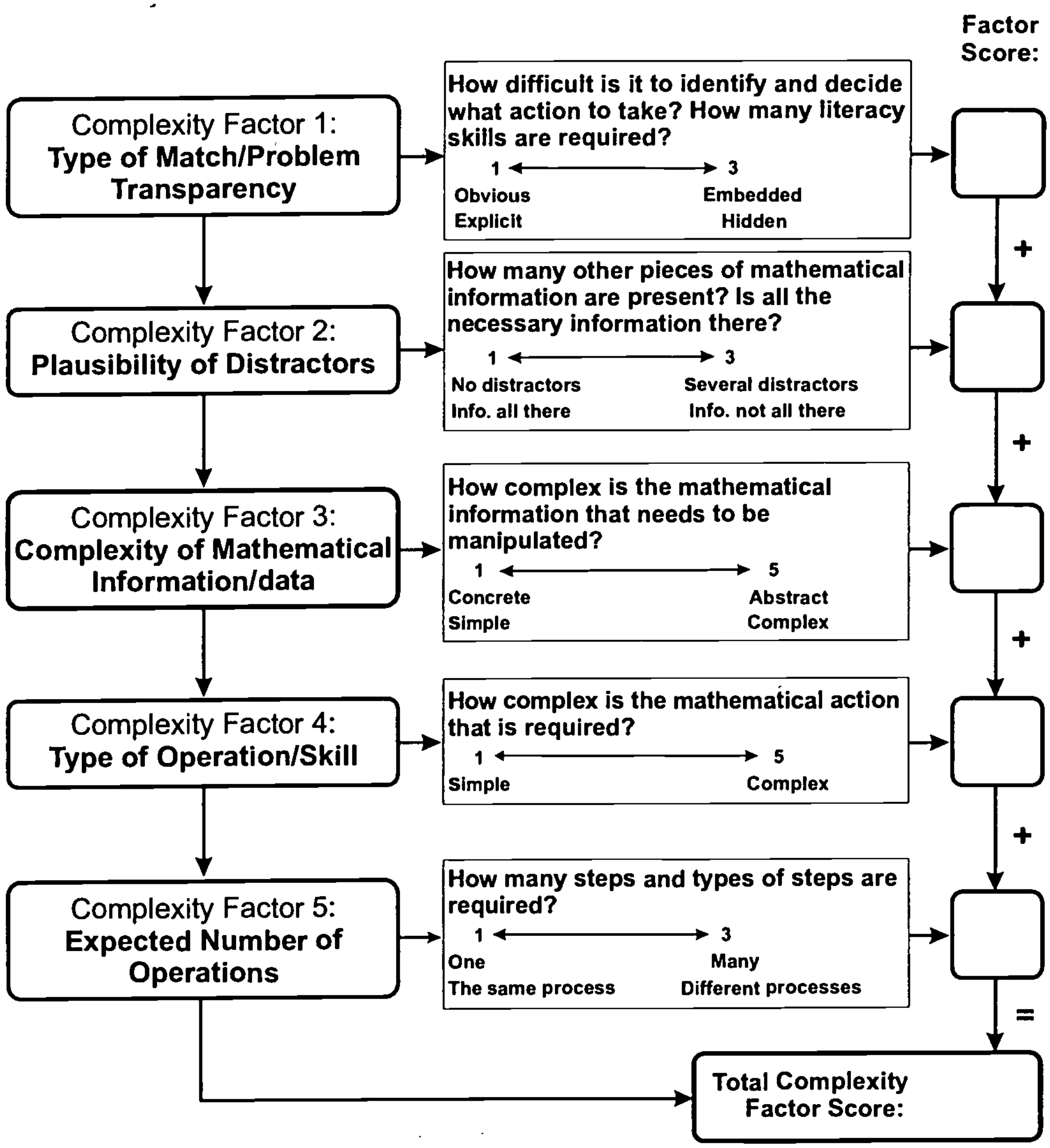
and requires activation of a range of enabling knowledge, behaviors, and processes
mathematical knowledge and understanding
mathematical problem-solving skills
literacy skills
beliefs and attitudes.

A comparison of these facets of numeracy with what was included in the previous concept and definition of quantitative literacy of the IALS demonstrates how far the ALL concept of numeracy goes above and beyond the QL Scale. While the IALS QL scale depended on stimuli that were text-based and often required considerable literacy skills, the numeracy scale includes stimuli that span the range from minimum text dependency to high. The IALS QL scale only assessed the application of arithmetic operations while the numeracy scale includes the assessment of a broad range of skills and knowledge.

Complexity Factors
A scheme of five factors was developed to account for the difficulty of different tasks, enabling an explanation of observed performance in terms of underlying cognitive factors.

These five factors are: (1) Type of match/problem transparency; (2) Plausibility of distractors (including in text); (3) Complexity of mathematical information/data; (4) Type of operation/skill; (5) Expected number of operations. These factors have been used to attempt to estimate, separately and in interaction, the difficulty level of the numeracy tasks. Each numeracy task or item that was developed for the test was given a rating between 5 and 19, the sum of the numbers assigned for each of the factors as outlined in the flow chart.

Diagram 1: Complexity Flow Chart



Each of these five factors has been developed in detail in the following tables.

Scoring for Each of the Complexity Factors

Complexity Factor 1. Type of match/Problem transparency How difficult is it to identify and decide what action to take? How many literacy skills are required?		
Score 1	**Score 2**	**Score 3**
In the question and the stimulus, the information, activity or operation required: - is clearly apparent and explicit—and all required information is provided - is specified in little or no text, using familiar objects and/or photographs or other clear, simple visualizations - is about locating obvious information or relationships only - closed question—not open-ended	In the question and the stimulus, the information, activity or operation required: - is given using clear, simple sentences and/or visualizations where some translation or interpretation is required - is located within a number of sources within the text/activity. - fairly closed question	In the question and the stimulus, the information, activity or operation required: - is embedded in text where considerable translation or interpretation is required - and/or - may need to be derived or estimated from a number of sources within or outside the text/activity - and/or - the information or action required is not explicit or specified - more complex, open-ended task

Complexity Factor 2. Plausibility of distractors How many other pieces of mathematical information are present? Is all the necessary information there?		
Score 1	**Score 2**	**Score 3**
- no other mathematical information is present apart from that requested—no distractors	- there is some other mathematical information in the task that could be a distractor - the mathematical information given or requested can occur in more than one place - may need to bring to the problem simple information or knowledge from outside the problem.	- other irrelevant mathematical information appears - mathematical information given or requested appears in several places. - necessary information or knowledge is missing, so outside information or knowledge needs to be brought in

Complexity Factor 5. Expected number of operations How many steps and types of steps are required?		
Score 1	**Score 2**	**Score 3**
one operation, action or process	application of two or three steps, the same or similar operation, action or process	integration of several steps covering more than one different operation, action or process

Complexity Factor 3. Complexity of mathematical information/answer required
How complex is the mathematical information that needs to be manipulated?

Score 1	Score 2	Score 3	Score 4	Score 5
Context Based on very concrete, real life activities, familiar to most in daily life.	Based on common, real life activities.	Based on real life activities, but less often encountered.	Based on real life activities but unfamiliar to most	Based on abstract ideas or unfamiliar activity in a context new to most.
Quantity Whole numbers to 1,000 Fractions, decimals, percents - benchmark fractions ($^1/_2$, $^1/_4$, $^3/_4$) - decimal fraction for a half only (0.5) and equivalent as a percentage (50%)	- large whole numbers including millions - other benchmark fractions, like $^1/_3$ and $^1/_{10}$ - common decimals, like 0.1, 0.25 to 2 decimal places - common whole number percents, like 25% and 10%.	- large whole numbers including billions - other fractions - decimals to 3 decimal places (other than money) - all whole number percents	- negative integers - all remaining fractions, decimals and percentages	- all remaining types of rational (and some irrational) numbers including directed numbers
Pattern and relationship - very simple whole number relations and patterns	- simple whole number rates and ratios - whole number relations and patterns	- rates and ratios - relations and patterns including written everyday generalizations	- complex ratios, relations, patterns - simple formula	- formal mathematical information such as more complex formulae, knowledge of relationships between dimensions or variables, etc
Measures/ Dimension/Space - standard monetary values - common everyday measures for length (whole units) - time (dates, hours, minutes) - simple, common 2D shapes - simple localised maps or plans (no scales)	- everyday standard measures for length, weight, volume, including common fraction and decimal units - common 3D shapes and their representation via diagrams or photos - common types of maps or plans with visual scale indicators	- other everyday measures (area included) including fraction and decimal values - more complex 2D and 3D shapes, or a combination of 2 shapes - area and volume formulae - common types of maps or plans with ratio type scales	- all kinds of measurement scales - complex shapes or combinations of shapes	
Chance/Data - simple graphs, tables, charts with few parameters and whole number values - simple whole number data or statistical information in text	- graphs, tables, charts with common data including whole number percents—whole number scales in 1s, 2s, 5s or 10s - data or statistical information including whole number percents	- graphs, tables, charts with more complex data (not grouped data) - more complex data or statistical information including common average, chance and probability values - scales - more complex whole number, fractional or decimal scales	- complex graphs, tables or charts including grouped data - complex data or statistical information including probabilities, measures of central tendency and spread	

Complexity Factor 4. Complexity of Type of operation/skill
How complex is the mathematical action that is required?

Score 1	Score 2	Score 3	Score 4	Score 5
Communicate no explanation - a single simple response required (orally, or in writing)	- no explanation - a simple response required (orally, or in writing	- simple explanation of a (level 1 or 2) mathematical process required (orally, or in writing) -	- explanation of a (level 3) mathematical process required (orally, or in writing)	- complex, abstract and generative reasoning or explanation required
Compute - a simple arithmetical operation (+, -, x, ÷) with whole numbers or money	- calculating common fraction, decimal fraction and percentages of values - using common rates (e.g. $/lb.); time calculations; etc - changing between common equivalent fraction, decimal and percent values, including for measurements e.g. $^1/_4$ kg = 0.250kg	- more complex applications of the normal arithmetical operations such as calculating with fractions and more complex rates, ratios, decimals, percentages, or variables - simple probability calculations	- applications of other mathematical operations such as squares, square roots, etc	- more advanced mathematical techniques and skills e.g. trigonometry
Estimate	- estimating and rounding off (when requested) to whole number values or monetary units	- estimating and rounding off to requested number of decimal places	- making a contextual judgement re whether a found answer is realistic or not and changing the answer to the appropriate correct rounded (but not necessarily mathematically correct) answer.	
Use formula/ model	- evaluating a given formula involving common operations (+, -, x, ÷)		- developing/creating and using straight forward formulae - using strategies such as working backwards or backtracking (e.g. 15% of ? = $255)	- generative reasoning - using and interpreting standard algebraic and graphical conventions and techniques
Measure - knowing common straight forward measures - naming, counting, comparing or sorting values or shapes	- visualizing and describing shapes, objects or geometric patterns or relationships - making and interpreting standard measurements using common measuring instruments	- using angle properties and symmetry to describe shapes or objects - estimating, making and interpreting measurements including interpolating values between gradations on scales - converting between standard measurement units within the same system	- calculating measures of central tendency and spread for non-grouped data - converting between non-standard measurement units within the same system - counting permutations or combinations	- converting between measurements across different systems
Interpret - locating/identifying data in texts, graphs and tables - orientating oneself to maps and directions such as right, left, etc	- reading and interpreting data from texts, graphs and tables - following or giving straight forward directions	- interpolating data on graphs - calculating distances from scales on maps	- generating, organising, graphing non-grouped data - extrapolating data - reading and interpreting trends and patterns in data on graphs, including slope/gradient	- generating, organising, graphing grouped data - calculating measures of central tendency and spread for grouped data

The development of a scale that attempts to predict the complexity/difficulty of numeracy tasks was one of the more exciting, and challenging, aspects of the project.

Results and Next Steps

Based on the conceptual framework, over 100 items were developed and tested in a feasibility study in the U.S. and the Netherlands. A pool of 80 items remains that includes tasks at diverse levels of difficulty and that covers key facets of the conceptual framework for numeracy. For those 80 items, we found that the theoretical factors that were predicted to account for task difficulty were strongly correlated with the observed difficulty of items. A scatter plot of the data shows a significant negative linear correlation with an r of -0.859. Thus, about 74% of the variation in performance on the 80 items is accounted for by the complexity factors.

Preliminary results therefore provide initial support for the content validity and the construct validity of the numeracy scale. More reliable data are expected from the pilot study and the main survey.

The next steps are to revise the complexity scheme in the light of feedback received to date, and to ask other interested teachers or researchers to rate items using the complexity scheme. Initially this could be to test the inter-rater reliability of the scheme on the 80 pilot study items. A further validation stage could be to test the scheme on new sets of items, and compare the theoretical predictions of difficulty levels with the actual performance of the items with sample respondents.

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